NUMERICAL INVESTIGATION OF NONUNIFORM FLOW AROUND A SPHERE WITHIN THE FRAMEWORK OF THE MODEL OF A VISCOUS SHOCK LAYER

N. N. Pilyugin and R. F. Talipov UDC 532.526.2

Numerical solution of the complete Navier–Stokes equations currently is very difficult because there is no effective computational algorithm, especially in the region of large Reynolds numbers. Therefore various simplified models of the Navier–Stokes equations have found wide use [1-3]. There is practical interest in calculating the flow field and the heat transfer near a blunt body in the presence of a large nonuniformity in the incident flow, which in many cases leads to the development of local detached zones on the windward part of the body. Data from experimental and theoretical investigations of the resistance, the heat transfer, and the gas-dynamic behavior of the flow have been studied [4] where one of the bodies is in a supersonic flow behind the other. Numerical calculations of continuous supersonic viscous flow around blunt bodies were generalized within the framework of the theory of a supersonic viscous shock layer both with and without the flow of material from the surface. The problem of continuous flow around a blunt body at moderate Reynolds numbers ($Re_\infty \lesssim 10^3$) was solved asymptotically [5]; expressions were given for the heat transfer coefficient and the friction, and a criterion was presented for attached flow.

Equations for the complete viscous shock layer, first obtained in [6], describe the flow in the shock layer to second-order in the reciprocal square root of the characteristic Re_∞. A multistep procedure in solving the boundary problem of the complete viscous shock layer was first used in [7]. A numerical global-iteration method was presented [8] in detail to solve the complete stationary equations for the viscous shock layer; the method reduces to a multistep procedure in which the form of the shock wave and the pressure field are refined in each iteration.

The system of equations for the supersonic viscous shock layer has a parabolic form. However, the problem retains a boundary character because of the unkown shape of the shock wave. If the shape of the discontinuity is considered known (for example, it coincides with the shape of the body), then the boundary problem can be solved with given initial conditions with a single-step procedure. Here we examine such an approach for the supersonic viscous shock layer equations. Another approach [10], with a refined shape of the shock wave, uses equations for a thin viscous shock layer in the case of a uniform incident flow.

Here a global iteration method [8] is used to calculate a continuous supersonic flow of a perfect flow around an axisymmetric blunt body. We study the suitability of the simplified Navier–Stokes equations [1-3] to describe the behavior of the continuous flow near the blunt body. The critical values of the nonuniformity parameters are refined, for which a transition occurs to a detached flow regime.

Numerical results, which were found in the framework of equations for the complete viscous shock layer, are compared with results of an earlier solution to the complete Navier–Stokes equations. The calculations are performed for a sphere for Mach numbers $M_\infty \gtrsim 6$ and $10^2 \le Re_\infty \le 10^5$, with a consideration of slippage both at the discontinuity and on the surface of the body.

1. FORMULATION OF THE PROBLEM AND DESCRIPTION OF THE NUMERICAL SOLUTION METHOD

We will examine the stationary continuous supersonic flow of a viscous perfect gas around a smooth blunt body. The profiles of parameters in the incident flow are written in a dimensionless form [4, 11]

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$$V_1(r)=1-a\exp(-br^2),\ p_1(r)=(\gamma M_\infty^2)^{-1},$$
$$\rho_1(r)=B/(1+C\{1+V_1^2(1-a)^{-2}\}),\ B=1+C\{1+(1-a)^{-2}\}.$$

Here rR_0 is the distance to the axis of symmetry (R_0 is the radius of curvature of the body at the critical point); V_1V_∞, $\rho_1\rho_\infty$, and $p_1\rho_\infty V_\infty^2$ are the velocity, density, and pressure in the incident flow; and M_∞, V_∞, and ρ_∞ are the Mach number, the dimensional velocity, and the dimensional density for $r \to \infty$. The parameters a, b, and C are determined by the degree of discontinuity of the incident flow. We examine the case of moderate and large values of $Re_\infty = \rho_\infty V_\infty R_0/\mu_\infty$ (μ_∞ is the viscosity in the incident flow) and large values of M_∞: $Re_\infty \gtrsim 10^2$, $M_\infty \gtrsim 6$, and $\mu_\infty = \mu(T_\infty)$.

An important parameter for supersonic flow around blunt bodies is the temperature of adiabatic damping: $T_0 = T_\infty[1 + (\gamma - 1)\ 0.5M_\infty^2]$, which determines the viscosity coefficient typical of a flow field in the neighborhood of the critical point. The typical Reynolds number for the problem is computed from the formula $Re_S = \rho_\infty V_\infty R_0/\mu_S$ ($\mu_S = \mu(T_0)$). The parameter of viscous hyperbolic similarity, $\varepsilon = Re_S^{-1/2}$, is a measure of the boundary layer thickness, which is of order ε, and also of the shock wave – $O(\varepsilon^2)$ in the limiting case of infinitely large M_∞ and Re_∞.

Vasil'evskii and Tirskii [8] give the stationary two-dimensional system of equations for the complete viscous shock layer in terms of variables related to the body surface. We go to new independent variables ξ and η and to new flow functions

$$\xi=x,\ \eta=\frac{1}{\Delta}\int_0^y \rho\bar{r}\,dy,\ \bar{r}=r/r_w=1+\frac{y\cos\alpha}{r_w},$$
$$\Delta=\int_0^{y_s}\rho\bar{r}\,dy,\ f=\psi(2\pi\rho_\infty V_\infty r_w\Delta\cos\alpha)^{-1}$$

where xR_0 and yR_0 are the corodinates along and normal to the body surface; r_wR_0 is the distance from the axis of symmetry to the body contour; α is the inclination angle of the body contour to the axis of symmetry, and y_S is the tail of the shock wave. The system of equations in terms of the variables ξ and η is presented in [8]. We will limit ourselves to deriving the boundary conditions at the shock wave more exactly. For moderately small values of Re_∞, the shock wave is smeared, so the structure of the transition region through the shock wave must be analyzed. If the discontinuity is small enough with respect to the shock layer, then this analysis can be omitted to simplify the Navier–Stokes equations, and the boundary conditions at the jump can be replaced by the generalized Rankine–Hugoniot relations which consider the slippage at the discontinuity [9]. In the nonuniform case, the boundary conditions are derived as in [7-9]. We omit the derivation and write the final formulas in the variables ξ and η for $\eta = 1$:

$$C_1\frac{\partial u}{\partial\eta}+u+(1-k_s)C_2\cos\alpha-V_1\cos\alpha-\frac{\cos^2\beta\cdot\cos^2\beta_s}{Re_\infty\rho_1V_1\sin\beta}\frac{dV_1}{dr}=0,$$
$$\frac{C_1}{\cos^2\beta_s\cdot Pr}\frac{\partial H}{\partial\eta}+H-1+\frac{V_\infty^2}{H_0}\left(1-\frac{1}{Pr}\right)\left\{C_1u\frac{\partial u}{\partial\eta}-\frac{V_1\cos^3\beta}{Re_\infty}\frac{dV_1}{dr}\right\}=0,$$
$$p_s=(\gamma M_\infty^2)^{-1}+(1-k_s)\rho_1V_1^2\sin^2\beta,$$
$$C_1=\frac{\mu\rho\bar{r}\cos^3\beta_s}{\Delta\,Re_\infty\rho_1V_1\sin\beta},\ C_2=\frac{\sin\beta\cdot\sin\beta_s}{\cos\alpha},$$
$$H_0=C_pT_0,\quad k_s=\frac{\rho_1}{\rho_s}=\frac{\gamma-1}{\gamma+1}+\frac{2}{(\gamma+1)\rho_1V_1^2M_\infty^2\sin^2\beta},$$
$$\beta_s=\beta-\alpha.$$

Here $V_\infty u \cos\alpha$ and $-V_\infty v$ are the components of the velocity vector along the x and y axes; $\rho_\infty V_\infty^2 p$, $T_0 T$, $\mu_0 \mu$, and λ, are the pressure, temperature, viscosity coefficient, and thermal conductivity of the gas; HH_0 is the total enthalpy, $\gamma = C_p/C_v$; Pr = 0.71 is the Prandtl number; β is the angle between the tangent to the shock wave surface and the axis of symmetry; the subscript s indicates parameters at the discontinuity.

In the general case of injection of gas from the surface of the body for $\eta = 0$, we use the condition

$$f_w = -\frac{Qr_w}{2\Delta \cos\alpha},\ u = 0,\ H = h_w,\ Q = \frac{1}{r_w^2}\int_0^{r_w} \frac{\rho_w v_w \, dr_w^2}{\sin\alpha}$$

where f_w is the flow function, and subscript w denotes parameter values on the body. In the case of moderately small Reynolds numbers (Re_∞ = 30-100), we consider slippage along the surface of the body and the temperature jump [7], which in terms of the variables η and ξ has the form

$$u = \frac{a_1\mu}{\mathrm{Re}_\infty \Delta}\sqrt{\frac{\rho}{p}}\left(\frac{\partial u}{\partial \eta} - \frac{\Delta}{\rho}\varkappa u\right),$$

$$H = h_w + \frac{b_1}{\Delta}\frac{\mu}{\mathrm{Re}_\infty}\sqrt{\frac{\rho}{p}}\frac{\partial}{\partial\eta}\left(H - \nu\frac{u^2V_\infty^2}{2H_0}\right) + \nu u^2 V_\infty^2/2H_0,\ \nu = \cos^2\alpha.$$

According to [7], a_1 = 1.2304 and b_1 = 2.3071. The output Δ from the discontinuity in terms of the new variables is determined from the mass balance condition:

$$\Delta = \frac{(1 + F + Q)\, r_w}{(f_s - f_w - I)\, 2\cos\alpha},$$

where

$$I = \int_0^1 \frac{d\eta}{\rho};\ F(r_s) = \frac{B(1-a)^2}{2bCr_w^2}\ln A;$$

$$A = \left|\frac{\alpha_0 - W_1}{\alpha_0 - W_0}\right|^{\alpha_0 - 1}\left|\frac{\alpha_0 + W_0}{\alpha_0 + W_1}\right|^{\alpha_0+1};$$

$$r_s = r_w + y_s\cos\alpha;\ W_1 = V_1(r_s);\ W_0 = 1 - a;\ \alpha_0 = (1-a)\times \times\sqrt{(1+C)/C}.$$

We also make use of the geometric relationship

$$\mathrm{tg}\,\beta_s = \frac{1}{H_{1s}}\frac{dy_s}{dx},\ H_{1s} = 1 + \varkappa y_s\,,$$

where κ is the curvature of the body contour.

Formulas from [8] and [12] are used as the initial approximation for the shape of the shock wave

$$\mathrm{tg}\,\beta_s = D\left(\sqrt{\mathrm{tg}^2\alpha + \frac{2D-1}{D^2}} - \mathrm{tg}\,\alpha\right),$$

$$D = \frac{R_{s0}}{2R_0 H_{1s0}},\ H_{1s0} = 1 + y_s(0)/R_0$$

or in a clearer form

$$D = \frac{1{,}05 + 1{,}65k_{s0}}{2(1 + 0{,}78k_{s0})},\ k_{s0} = \rho_\infty/\rho_{s0}.$$

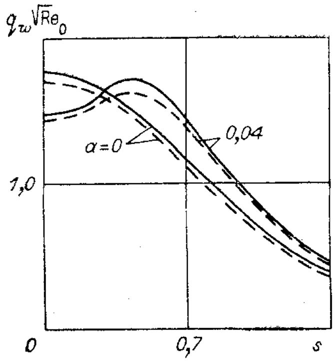

Fig. 1

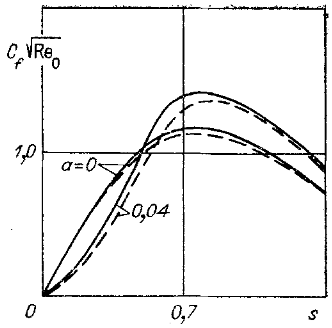

Fig. 2

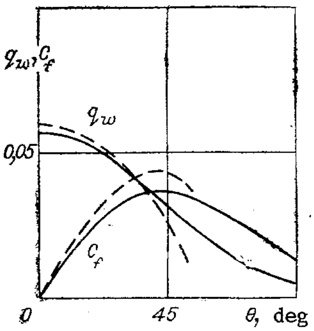

Fig. 3

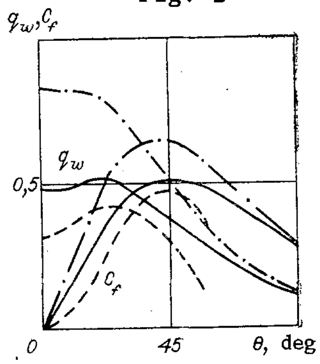

Fig. 4

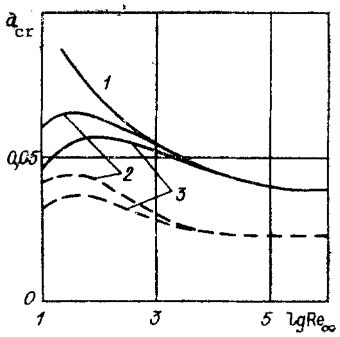

Fig. 5

The calculations were conducted over a wide range of parameters for the incident flow: $M_\infty \geq 6$; $30 \leq Re_\infty \leq 10^6$; $0.01 \leq T_w \leq 1$; and $f_w = 0$. In most cases b = 7.2; C = 3.0; the parameter a is examined in the interval $0 \leq a \leq a_{cr}$, where a_{cr} (b, C, M_∞, Re_∞, T_w) is the critical value at which detached flow occurs in the shock layer at any part of the body. Calculations were performed on nonuniform grids. Up to 30 nodes of the grid were specified perpendicular to the shock layer, depending on Re. Approximately half of these points were in the boundary layer. For flow around the sphere, 24 intervals were examined along the longitudinal coordinate ξ with extra intervals near the critical point. The minimum step in the central angle θ was around 2°.

The numerical solution made use of a method with enhanced accuracy, analogous to the one in [13]. The nonuniform case was calculated as follows. The nonuniformity parameters b and C were fixed, but a was assigned a step (usually 0.005 or 0.01). The uniform case (a = 0) was calculated to convergence. Then a was increased by the step value and global iterations were repeated to convergence. Afterwards a again was increased and the process was repeated. Here the form f the shock wave and the pressure gradient distribution for a smaller a were used as initial approximations to obtain the solution. The total calculation was conducted to the maximum possible value of a. Each new value of a required 10-15 global iterations for a step of 0.01 and 5-8 iterations for a step of 0.005.

After the difference equations were solved, the distribution of the thermal flux q_w (relative to $\rho_\infty V_\infty H_0$) and the coefficient of frictions C_f With the friction force relative to $0.5\rho_\infty V_\infty^2$) were determined from the formulas

$$q_w = \frac{\mu\rho}{\Pr \mathrm{Re}_\infty} \frac{\partial H}{\partial \eta} \quad (\eta = 0), \; C_f = \frac{2 \cos \alpha \mu\rho}{\mathrm{Re}_\infty} \frac{\partial u}{\partial \eta} \quad (\eta = 0).$$

2. CALCULATED NONUNIFORM SUPERSONIC FLOW AROUND A SPHERE

We examine the calculated results of the local thermal and dynamic characteristics for laminar flow around a sphere.

Figures 1 and 2 compare data obtained from the complete viscous shock layer (solid curves) and the complete Navier—Stokes equations [14] (dashed curves) for flow around a cooled sphere. The results are presented in terms of the dimensionless variables used in [14]; s is the dimensionless coordinate along the surface of the body. Ahead of the shock wave at the axis of symmetry, $M_0 = 6$, and $\mathrm{Re}_0 = 177$. The temperature factor, defined as the ratio T_w/T_0, has a value of 0.15; b = 7.2; C = 3.0; Pr = 0.7; the viscosity coefficient is determined from the power law $\mu \sim \sqrt{T}$; and the ratio of specific heats $\gamma = 1.4$. Figures 1 and 2 compare the distributions of the thermal flux coefficient $q_w\sqrt{\mathrm{Re}_0}$ and the friction $C_f\sqrt{\mathrm{Re}_0}$. The maximum difference in the nonuniform case between the solid and dashed curves of the thermal flux is 3%, and less than 5% for the friction. Also, Golovachev and Leont'eva [14] show that the results are presented on the friction and the thermal flow have an accuracy near 5%. We compared the value of $q_w\sqrt{\mathrm{Re}_0}$, calculated from the model of the complete viscous shock layer, versus Re_0 with calculations for the complete Navier—Stokes equations [15] for $\theta = 0$ and 30°. We found that as Re_0 increased, the agreement worsened somewhat with increasing a, which is explained by the closeness of the nonuniformity parameters to their critical values. However the maximum difference in the curves for the critical Re values does not exceed 9%. The results show that the basic aerodynamic characteristics (the thermal flow and friction), which are determined by the complete viscous shock layer model, have satisfactory accuracy over the whole investigated range of nonuniformity parameters.

Figures 3 and 4 compare data obtained from equations for the complete viscous shock layer (solid and dot-dash curves) and equations for the supersonic viscous shock layer [4] (dashed curves) for $M_\infty = 20$, Pr = 0.7, $\gamma = 1.4$, and $T_w = 0.1$. Figure 3 shows the thermal flow and friction in the uniform case (a = 0) for $\mathrm{Re}_\infty = 10^4$, and Fig. 4 for nonuniform flow (a = 0.4 and $\mathrm{Re}_\infty = 10^2$). The solid curves correspond to a calculation that considers slippage effects at the discontinuity and the sticking conditions at the surface of the body. For $\mathrm{Re}_\infty = 10^2$, the calculations were conducted with slippage effects both at the discontinuity and at the surface of the body (this curve is not shown to avoid complicating the figure) and without considering these effects (dot-dash curves). Comparison of the calculations using the supersonic model (slippage only at the discontinuity) with those using the complete model shows that for a uniform incident flow the difference in the thermal flux near the critical point (approximately to 30°) does not exceed 5-8% for all Re values. The agreement in the friction coefficient is somewhat better at small Re and worse at large values and becomes much worse in the nonuniform case. The thermal flow is reduced by 35% at the critical point for the supersonic model for $\mathrm{Re}_\infty = 10^2$ and by 20% for $\mathrm{Re}_\infty = 10^4$. The agreement improves somewhat on part of the side surface of the sphere (approximately at 30°). The difference in the friction coefficient is also large in the nonuniform case (it is almost 20% for example, at $\mathrm{Re}_\infty = 10^4$ for the maximum values of the friction coefficient).

Slippage effects at the discontinuity (Figs. 3 and 4) are much larger for friction and the thermal flow than for slippage and the temperature jump at the surface. Calculations without slippage effects lead to doubled values of the thermal flow at the critical point, and the friction coefficient is increased by 30% (Figs. 3 and 4). With slippage only at the discontinuity, the thermal flow increases by 17% and the friction coefficient by 10%. Evaluations close to these are also obtained in [7] for uniform flow. From Figs. 3 and 4 it can be seen that slippage effects at the discontinuity appear basically near the critical point, where the temperature behind the discontinuity is close to the deceleration temperature. The effects of slippage at the surface occur all over the sphere. In the nonuniform case (Fig. 4) they are approximately the same as in the uniform case (Fig. 3).

Figure 5 shows the a_{cr} versus Re_∞ for parameter values corresponding to Fig. 3. Curve 1 is the calculation from the complete viscous shock layer model with slippage at the surface and an arbitrary temperature $0.1 \le T_w \le 1$; the solid curves 2 and 3 are the calculation using the total model with sticking conditions for $T_w = 0.1$ and 0.3; the dashed curves 2 and 3 are the corresponding calculations for the supersonic model, which give somewhat reduced values of a_{cr} over the whole range of Re values. At large Re values, the function $a_{cr}(Re_\infty)$ becomes constant. Considering the sticking conditions (curves 2 and 3) leads to a branching of the curves as a function of the wall temperature and a nonmonotonic behavior of $a_{cr}(Re_\infty)$ in the range of small Re_∞ [4]. The calculation with slippage conditions at the surface does not display that splitting, and a_{cr} grows monotonically as Re decreases (curve 1). We note that for $Re < 10^2$ and $M_\infty \approx 20$, the Knudsen number becomes of order 1, and application of the equations of continuum mechanics requires additional research. Therefore the results for small Re_∞ in Fig. 5 must be considered qualitative and require refinement in the transition region to the rarefied gas model. Comparison of the solid curves 1-3 shows that, as the wall is cooled, the influence of slippage at the surface decreases, which agrees with the estimates [9] for uniform flow.

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